

# An Interval Arithmetic-Based Methodology For Reliable Power Flow Analysis With Of Data Uncertainty

G. Sandhya Rani<sup>1</sup>, T. Srinivasa Rao<sup>2</sup>

<sup>1</sup> Vasavi College of Engineering, Department of Electrical and Electronics Engineering, Hyderabad, India.

<sup>2</sup> Avanti institute of Engineering and Technology, Department of Electrical and Electronics Engineering, Narsipatnam, India.

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**ABSTRACT :** In this research paper, the authors have developed an Interval Improved Fast Decoupled Power Flow (IIFDPF) algorithm to address data uncertainty in load and generation for power systems. The algorithm utilizes Interval arithmetic-based techniques and solves the Interval power flow method. The objective of this study is to handle uncertainties arising from measurement errors by accurately modeling load and generator bus data. To achieve this, the authors incorporate Interval arithmetic-based techniques into the IIFDPF algorithm, which enables the treatment of bus data uncertainty. The algorithm employs Interval Newton's method to solve the nonlinear model and two sets of linear Interval equations, namely the decoupled active power (P) and reactive power (Q) equations. In each iteration, the algorithm updates the voltage angle and bus voltage using different strategies, and the Newton operator is utilized for solving these equations. The proposed method demonstrates faster convergence and saves computing time compared to traditional probabilistic Monte Carlo methods. To validate the effectiveness of the proposed method, the authors conducted tests on IEEE-30, 57, and 118 bus systems. The results obtained from the proposed method were compared with those obtained from the traditional probabilistic Monte Carlo method. The comparison confirmed that the proposed method achieved faster convergence and validated its effectiveness. Additionally, the authors discussed the drawbacks of existing interval power flow methods in the paper.

**Keywords:** Load Flow Studies, Y-matrix and Z-matrix iteration, Newton-Raphson method, Fast decoupled method, Fuzzy logic, Interval arithmetic, Probabilistic methods.

## I. INTRODUCTION

In recent years, researchers have devoted significant effort to the field of power flow analysis. However, conventional methodologies fail to address the presence of uncertainties in the mathematical modeling of power systems. Uncertainties in power systems can arise from various sources, such as environmental factors, regulatory changes, and technological advancements. The uncertainties in power systems can be attributed to several factors.

Firstly, the type of mathematical model used can introduce uncertainties. Different assumptions and simplifications made in the modeling process can lead to uncertainties in the results. Secondly, uncertainties can arise from the representation of various physical

components within the power system. Inaccuracies in modeling these components can affect the overall system behaviour. Thirdly, uncertainties can stem from errors in the parameter values used in the models. Inaccurate or imprecise parameter values can introduce uncertainties in the analysis. Fourthly, uncertainties can be introduced through the introduction of noise at the inputs of the system. Finally, numerical modeling using finite arithmetic can also contribute to uncertainties in the results.

To represent uncertainties, researchers have explored both qualitative and quantitative approaches. Qualitative uncertainties are typically expressed in verbal terms, such as "near to" or "smaller than." On the other hand, quantitative uncertainties are represented numerically using mathematical functions with deterministic parameters. There are three main approaches to address uncertainties in power systems: probabilistic methods, fuzzy logic, and interval arithmetic (IA). Probabilistic methods use probability theory to handle uncertainties, fuzzy logic employs fuzzy variables to represent uncertainties, and interval arithmetic utilizes interval variables to capture uncertainties. However, it is worth noting that the available methods for handling uncertainties in power systems are still limited in number. In conclusion, uncertainties in power systems pose challenges to conventional power flow analysis methodologies. The sources of uncertainties can vary, and both qualitative and quantitative approaches have been explored to represent them. However, the methods for handling uncertainties in power systems are relatively scarce, and further research is needed in this area.

### **Uncertain Power Flow Study:**

#### **Probabilistic load flow:**

The concept of probabilistic load flow was introduced over 30 years ago [1-2], and since then, extensive research has been conducted on various methods [3-17]. Probabilistic load flow methods can generally be categorized into three groups: simulation methods, analytical methods, and approximate methods [17, 18].

One commonly used simulation method is Monte Carlo simulation [19], which has been employed in reliability assessments for many years. This approach involves performing load flow calculations multiple times based on samples of uncertain factors. By statistically analyzing the results, probability distributions of reliability indices and system states of interest can be obtained. However, Monte Carlo simulation is time-consuming, and although various variance reduction techniques [20] have been employed to mitigate this issue, a significant number of calculations (tens of thousands) are still required to obtain meaningful results. Consequently, the high computational cost limits the practical application of Monte Carlo simulation primarily to long-term expansion planning. On the other hand, analytical methods offer computational advantages compared to simulation methods. These methods are more efficient in terms of computational requirements and provide quicker results.

In the initial stages of analytical methods, convolution techniques [2-3] were commonly used to derive probability distributions for desired variables. However, these methods still had limited computational efficiency, although attempts were made to improve them by employing fast Fourier transform [5]. As a result, alternative analytical methods based on numerical characteristics such as moments and cumulant were developed [10-14]. These methods make

a fundamental assumption of independent uncertain inputs and often utilize linearized load flow equations to leverage the properties of these numerical characteristics. Experimental results have demonstrated that linearized load flow equations remain valid within a specific range of uncertain inputs [10, 12], allowing for significant computational efficiency. The cumulant method [12-14] represents one such approach. However, linearization of load flow equations necessitates a fixed network configuration, which presents challenges when dealing with network changes, such as branch outages [13, 14]. Therefore, addressing network modifications, such as branch outages, requires the implementation of complex techniques in these analytical methods.

Approximate methods offer a compromise between speed and precision, and two well-known approaches in this category are the first-order, second-moment method [15] and the point estimate method [16]. While approximate methods also utilize numerical characteristics of inputs like analytical methods, they employ different strategies to determine the numerical characteristics of outputs, without relying on linearized load flow equations. For instance, the point estimate method [16] maps uncertain inputs to specific locations and corresponding weights, enabling straightforward calculation of output moments. However, these methods have certain limitations. Non-normal probability distributions, statistical dependence among input variables, and accurately identifying probability distributions for certain input data (e.g., power generated by wind or solar generators) pose challenges. These complexities can result in computationally intensive procedures, which may restrict the practical application of these methods, particularly when dealing with large networks.

Hence, despite their advantages in terms of computational efficiency, approximate methods face challenges due to non-normal distributions, statistical dependencies, and accurately characterizing certain input data distributions. These limitations may hinder their widespread use in practical applications, especially when studying large-scale networks.

### **Fuzzy load flow:**

Zadeh Lotfi extensively documented the terminology associated with Fuzzy Inference Systems, including key terms such as fuzzification, rule base, membership function, linguistic variable, and defuzzification [21]. Fuzzy Logic (FL) has found applications in various power system problems, including load forecasting, system control, security assessment, system planning, and power system stability [22-23]. In recent years, fuzzy logic-based approaches have been utilized in different ways to address power flow problems [24-28].

One study [24] applied the principles of Fuzzy set theory to model input parameters for power flow analysis. This approach leveraged Fuzzy logic to handle the uncertainties and imprecise nature of power system variables. Another work [25] focused on utilizing Fuzzy logic for on-line voltage estimation in situations such as outages and load changes. By employing Fuzzy-based techniques, these methods aimed to enhance the accuracy and robustness of voltage estimation in dynamic power system conditions.

These examples demonstrate how Fuzzy logic has been effectively employed in power system analysis and control, specifically in power flow analysis and voltage estimation under varying operating conditions.

In the context of power systems, a Fuzzy set-based reasoning approach was developed for contingency ranking in [26]. Another application of Fuzzy logic was demonstrated in [27], where parameters such as transmission-line impedance, phase angle, and transformer tap positions were adjusted using Fuzzy logic techniques.

In [28], real and reactive power mismatches per voltage magnitude at each bus of the system were selected as crisp input values. These values were then fuzzified using a fuzzifier. A rule base, defined by the process logic, was utilized to process the Fuzzy output signals, which were subsequently defuzzified. The resulting crisp values were employed to correct the voltage angle and magnitude at each bus of the system. Triangular membership functions were used for the purpose of fuzzification.

Furthermore, Fuzzy logic has been applied in load flow studies and contingency ranking to adjust variable parameters [29]. By leveraging Fuzzy logic techniques, these studies aimed to enhance the accuracy and efficiency of load flow analysis and contingency assessment in power systems.

### **Interval load flow:**

In recent years, there has been a growing trend in representing uncertain variables in load flows using Interval numbers. Interval arithmetic allows for obtaining solutions that can be associated with every possible value within a given range, providing uniform validity. This appealing characteristic of Interval arithmetic has attracted the attention of researchers, leading to significant contributions in solving uncertain electrical power flow problems. Researchers such as Barboza, Zian Wang, and others have made notable contributions to the international literature in this field [30-36]. Their work focuses on utilizing Interval arithmetic to address the challenges posed by uncertainty in electrical power flow analysis. These efforts have aimed to enhance the accuracy and reliability of load flow calculations by considering the range of possible values for uncertain variables, thereby providing more robust solutions. The contributions of these researchers have advanced the understanding and application of Interval arithmetic in solving uncertain electrical power flow problems, making a significant impact in the international literature.

Zian Wang and F.L. Alvarado [30] proposed a method for solving load flow problems using Interval arithmetic, which takes into account the uncertainty associated with nodal values. They suggested that the non-linear equations can be solved using operators such as Interval Newton, Krawczyk, or Hansen-Sengupta. These operators facilitate obtaining the required solutions for the non-linear equations in the context of Interval arithmetic.

Barboza and other researchers presented their methodology for solving uncertain power flow problems through a series of research articles [32-36]. They also applied Interval mathematics to load flow analysis [33-35], utilizing Krawczyk's method to solve non-linear

equations. They noted that Krawczyk's method helps overcome the issue of excessive conservatism encountered in solving Interval linear equations. To ensure convergence, these methods precondition the linearized power flow equations using an M-matrix.

The research contributions of Zian Wang, F.L. Alvarado, Barboza, and others have demonstrated the application of Interval arithmetic and specific operators to solve load flow problems with uncertainty. By incorporating uncertainty considerations and employing appropriate mathematical techniques, these methods offer potential solutions to uncertain power flow analysis, overcoming challenges such as excessive conservatism and guaranteeing convergence through preconditioning. In reference [30], the set of non-linear equations was solved using the Gauss-Seidel method. However, preconditioning is necessary, and there is no guarantee of convergence if the Interval input is too large. Therefore, this method cannot provide an exact solution, and its convergence may be limited in certain cases.

In reference [37], Interval arithmetic was employed in the Fast Decoupled power flow method to obtain solutions for power flow problems with uncertainty. Linearization was achieved through the Interval Gauss elimination method. It is important to note that the use of Interval Gauss elimination in the power flow process yields realistic solution bounds only for certain special classes of matrices. However, this approach often exhibits excessive conservatism, resulting in conservative solution bounds that may be overly cautious or restrictive. While these methods utilize Interval arithmetic to address uncertainty in power flow analysis, they have limitations. The Gauss-Seidel method may not guarantee convergence for large Interval inputs, and the solution may not be exact. Similarly, the Interval Gauss elimination method used in Fast Decoupled power flow can lead to conservative solution bounds, which may restrict the range of feasible solutions. These considerations highlight the challenges associated with using Interval arithmetic in power flow analysis and the need for further research to develop more accurate and efficient methods for handling uncertainty in power systems.

The recent methods mentioned above for the Interval Power Flow method, considering uncertainty, have several limitations:

1. The Interval Power Flow method requires solving non-linear equations, which can be done using operators such as Interval Newton, Krawczyk, or Hansen-Sengupta. This adds complexity to the computation.
2. The problem of excessive conservatism is addressed by Krawczyk's operator, but it requires preconditioning with the Jacobian matrix during the linearization process. Computing the inverse of the Jacobian matrix remains a computationally intensive task.
3. The use of Interval Gauss elimination in the power flow solution process yields realistic solution bounds only for specific classes of matrices, such as M-matrices, H-matrices, diagonal-dominant matrices, and tridiagonal matrices.

4. Due to the nature of interval arithmetic, the width of the interval components tends to grow larger as interval calculations are carried out in the Gaussian elimination process.
5. If a solution is obtained using Interval Gauss elimination, it is likely that the width of the components is very large. Additionally, there are cases where all contenders for pivot, or the bottom right elements in the upper triangular system, contain zero, leading to a breakdown of the algorithm due to division by zero.
6. Interval Gauss-Seidel iteration is used to solve the interval equations, but convergence is guaranteed only when the linearized power flow equation is preconditioned by an M-matrix.
7. Despite their limitations, Interval algorithms can be faster than conventional algorithms.

These limitations highlight the challenges and complexities associated with utilizing Interval arithmetic in power flow analysis with uncertainty. Further research is needed to develop more efficient and accurate methods that can overcome these limitations and provide reliable solutions for power system analysis.

The computational properties of different methods for handling uncertainty can vary significantly, with some methods being more time-consuming than others. By understanding the relationships among these methods, we can choose a faster yet still adequate approach and draw valid conclusions.

Among the methods, interval mathematics (IM) stands out as the simplest and most widely used. It enables numerical computations by representing each quantity as an interval of floating-point numbers, without incorporating a probability structure. Given its simplicity and popularity, it is worthwhile to explore additional techniques within the realm of interval mathematics that are suitable for large-scale computations.

Investigating and developing further techniques within interval mathematics can lead to improved efficiency and scalability, making it feasible to apply interval-based approaches to larger and more complex systems. By leveraging the advantages of interval mathematics and exploring innovative techniques, researchers can advance the computational capabilities of uncertainty handling methods, making them more suitable for practical applications and large-scale computations.

## **II. Interval Improved Fast Decoupled Power Flow (IIFDPF) Under Uncertainty:**

Firstly, interval numbers are used to express the uncertain variables in power system such as the uncertainty of all the load bus (PQ bus i.e.  $P_d$  and  $Q_d$ ) and generator bus (PV bus i.e.  $P_g$  and  $Q_g$ ).

Real and reactive powers can be expressed as

$$P_p = |V_p| \sum_{q=1}^n |V_q| |y_{pq}| \cos(\theta_{pq} + \delta_q - \delta_p) \quad p = 1, 2, \dots, n \quad (1)$$

$$Q_p = -|V_p| \sum_{q=1}^n |V_q| |y_{pq}| \sin(\theta_{pq} + \delta_q - \delta_p) \quad p = 1, 2, \dots, n \quad (2)$$

Conventional Power flow equations (1) and (2) become interval power flow equations by introducing uncertainty in the parameters as in (3) and (4) respectively.

Interval power flow equations as below by introducing uncertainty in the parameters.

$$[P_{pcal}^-, P_{pcal}^+] = [\tilde{V}_p] \sum_{q=1}^n \{[\tilde{V}_q] |Y_{pq}| \cos(\theta_{pq} + \tilde{\delta}_q - \tilde{\delta}_p)\} \quad (3)$$

$$p = 1 \dots \dots n ; p \neq \text{slack}$$

$$[Q_{pcal}^-, Q_{pcal}^+] = [\tilde{V}_p] \sum_{q=1}^n \{[\tilde{V}_q] |Y_{pq}| \sin(\theta_{pq} + \tilde{\delta}_q - \tilde{\delta}_p)\} \quad (4)$$

$$p = 1 \dots \dots n ; p \neq PV \quad p \neq \text{slack}$$

Where,  $n$  is the number of buses

$$[\theta_{pq}^-, \theta_{pq}^+] = [\theta_p^-, \theta_p^+] - [\theta_q^-, \theta_q^+]$$

The interval power flow method essentially means a procedure to find a solution for the following interval power equations i.e., solving (3), (4) for  $[V_p^-, V_p^+]$  where ( $p \in PQ$  buses) and  $[\delta_p^-, \delta_p^+]$  where ( $p \neq \text{slack}$ ) for given  $[P_p^-, P_p^+]$  where ( $i \neq \text{slack}$ ),  $[Q_p^-, Q_p^+]$  where ( $p \in PQ$  buses) and  $[V_p^-, V_p^+]$  where ( $p \in PV$  buses).

$\tilde{V}_p$  and  $\tilde{\delta}_p$  can be calculated by:

$$\tilde{V}_p \text{ or } V_{p0} = [V_p^- + V_p^+]/2 \quad \text{and} \quad \tilde{\delta}_p \text{ or } \delta_{p0} = [\delta_p^- + \delta_p^+]/2$$

it must be noted here that equations (3) and (4) differ from the standard load flow equations in polar coordinates. Since the active and reactive power at all the PQ buses and active power and voltage magnitude at all the PV buses are intervals.

### Solving Interval Improved Fast Decoupled Power Flow (IIFDPF)

#### Initialization of iterative process

The IIFDPF method starts after convergence of deterministic or punctual fast decoupled power flow FDPF method. Its initialization carried out based on deterministic or punctual voltage profile and on definition of load variations is as follows:

The real and reactive powers are given by

$$P_{psp} = P_{g_p} - P_{d_p} \quad \text{and} \quad Q_{psp} = Q_{g_p} - Q_{d_p},$$

Where  $P_{g_p}$  and  $Q_{g_p}$  are the generated real and reactive powers at bus  $p$ , and  $P_{d_p}$  and  $Q_{d_p}$  are the real and reactive power loads at bus  $p$ , respectively. Assuming the percent of uncertainty is 'e'.

$$[P_{psp}^-, P_{psp}^+] = [P_{psp}(1 - e), P_{psp}(1 + e)]$$

5(a)

$$[Q_{psp}^-, Q_{psp}^+] = [Q_{psp}(1 - e), Q_{psp}(1 + e)] \quad 5(b)$$

Where  $P_{psp}$  and  $Q_{psp}$  are specified active and reactive powers at bus  $p$  respectively, obtained from given bus data for the given test system. In operation of actual power system, the influence of parameter uncertainty of electric lines and transformer factor is not often small enough to be neglected.

Interval voltages are initialized by using the deterministic or punctual voltage profile as midpoint and the largest load data variation factor i.e. uncertain error as radius of interval. Thus:

$$[V_p^-, V_p^+] = [V_{po}(1 - e), V_{po}(1 + e)] \quad 5(c)$$

$$[\delta_p^-, \delta_p^+] = [\delta_{po}(1 - e), \delta_{po}(1 + e)] \quad 5(d)$$

where  $V_{po}$  and  $\delta_{po}$  are obtained from deterministic or punctual load flow in order to ensure a good initial condition for convergence of iterative process. Where equations 5(a),5(b) are interval active and reactive powers and equations 5(c) and 5(d) are the voltage magnitude and voltage phase angle in interval model.

The Fast Decoupled Power Flow Method (FDPFM) is one of the improved methods; which was based on a simplification of the Newton –Raphson’s method and reported by Stott and Alsac in 1974. This method due to its calculation simplifications, fast convergence and reliable results, becomes widely used in power flow analysis, after simplifications and assumptions of NR method.

The assumptions valid in normal power system operation are follows:

$$\cos \delta_{pq} \cong 1, \sin \delta_{pq} \cong 0,$$

$$G_{pq} \sin \delta_{pq} \ll B_{pq}; \text{ and } Q_p \ll B_{pp}|v_p|^2$$

With these assumptions, the entries of the [H] and [L] sub matrices, which are the elements of Jacobian matrix, become considerably simplified, as

$$H_{pq} = L_{pq} = -|V_p| |V_q| B_{pq}, p \neq q$$

$$H_{pp} = L_{pp} = -B_{pp} |V_p|^2$$

Matrices [H] and [L] are square matrices with dimensions  $(n-1)$  and  $(m-1)$ , respectively. Where  $(m-1)$  = number of PQ buses, and  $n-1$  = number of PQ and PV buses.

$$\frac{\Delta P_p}{|V_p|} = \sum_{q=2}^n [B_{pq}] \Delta \delta_q \quad p=2, \dots, n \quad (6)$$

$$\frac{\Delta Q_p}{|V_p|} = \sum_{q=2}^n [-B_{pq}] \Delta |V_q| \quad p=2, \dots, m \quad (7)$$

$$\frac{\Delta P_p}{|V_p|} = -B' \Delta \delta \quad (8)$$

$$\frac{\Delta Q_p}{|V_p|} = -B'' \Delta |V| \quad (9)$$



Then, the mathematical model of the Fast Decoupled Power Flow using interval arithmetic modelled as follows.

$$\frac{[\Delta P^-, \Delta P^+]}{[V_p^-, V_p^+]} = - [B]' [\Delta \delta^-, \Delta \delta^+] \quad (10)$$

$$\frac{[\Delta Q^-, \Delta Q^+]}{[V_p^-, V_p^+]} = - B'' [\Delta V^-, \Delta V^+] \quad (11)$$

The above two equations are in interval form which are developed from punctual equations discussed above.

The equations (10) & (11) can be simplified as below:

$$[\Delta \delta^-, \Delta \delta^+] = - [B]' \setminus \frac{[\Delta P^-, \Delta P^+]}{[V_p^-, V_p^+]} \quad (12)$$

$$[\Delta V^-, \Delta V^+] = - [B]'' \setminus \frac{[\Delta Q^-, \Delta Q^+]}{[V_p^-, V_p^+]} \quad (13)$$

Equations (12) and (13) involve two matrices, B' and B'', which are both real and sparse. These matrices have the structure of [H] and [L], respectively. As they only consist of admittances, their values remain constant throughout the load flow analysis. Therefore, they only need to be triangularized once at the beginning of the analysis. During the load flow analysis, separate convergence tests are conducted for real power and reactive power mismatches. These tests assess the accuracy of the calculated values compared to the specified target values. By applying separate convergence tests for real and reactive power, the load flow analysis can ensure the accuracy of both components of power flow. Overall, the load flow analysis involves triangularizing the B' and B'' matrices, which have a sparse structure and consist of constant admittance values. The analysis then applies separate convergence tests for real and reactive power mismatches to verify the accuracy of the calculated values.

The Interval Interval Decoupled Power Flow (IIFDPF) algorithm is employed to solve the interval power flow method. This method involves solving a resulting non-linear model using Interval Newton's operator, along with two sets of linear interval equations known as the Interval Decoupled P equations and Q equations, represented by equations (10) and (11) respectively. In contrast to the conventional Fast Decoupled method, where both the voltage magnitude and voltage angle are updated simultaneously in each iteration, the proposed IIFDPF algorithm takes advantage of the weak coupling between  $\Delta P$  and  $\Delta V$ , as well as between  $\Delta Q$  and  $\Delta \delta$ . This approach involves updating either the voltage angle or the voltage magnitude at each bus. Then, the real and reactive power values are recalculated, and the second variable is updated based on the first one that was updated. To improve speed and convergence reliability, the algorithm repeats the update of one variable multiple times while keeping the other variable at its last calculated value. This technique effectively reduces the number of iterations required, leading to faster convergence of the algorithm. Overall, the IIFDPF algorithm offers an improved approach for solving the interval power flow problem. By leveraging decoupling and selective variable updates, it achieves faster convergence and enhanced computational efficiency compared to traditional methods.

In order to solve the above equations (10) and (11) Newton operator needs to be calculated. Linear equation can solved as to give Newton's operator in equation (14)

$$N(\tilde{x}, x) = \tilde{x} - J(x)^{-1} f(\tilde{x}) \quad (14)$$

Using, the following notation  $\Delta x = -J(x)^{-1} f(\tilde{x})$  that results in the iteration, as equation(15),(16),(17)

$$\Delta x^{(k)} = -J(x^{(k)})^{-1} f(\tilde{x}^{(k)}) \quad (15)$$

$$N(\tilde{x}^{(k)}, x^{(k)}) = \tilde{x}^{(k)} + \Delta x^{(k)} \quad (16)$$

$$x^{(k+1)} = x^{(k)} \cap N(\tilde{x}^{(k)}, x^{(k)}) \quad (17)$$

from the above procedure, after calculating the Newton's operator, a new interval voltage solution is obtained as

$$[Vp^-, Vp^+] = [\tilde{V}p + \Delta V^-, \tilde{V}p + \Delta V^+] \text{ and } [\delta p^-, \delta p^+] = [\tilde{\delta} + \Delta\delta^-, \tilde{\delta} + \Delta\delta^+].$$

The IIFDPF algorithm employs a specific strategy where either the voltage angle ( $\delta$ ) or the voltage magnitude ( $V$ ) at each bus is updated, followed by a recalculation of the real and reactive power. The second variable is then updated based on which variable was updated first. This process is carried out in various combinations, categorized by the number of loops for each variable update. For instance, if the voltage angle is updated twice (2) and then the voltage magnitude is updated once (1) within the same iteration, it is denoted as (2:1). To assess the convergence of the proposed method, the algorithm calculates the difference between the radii at iteration ( $k + 1$ ) and the radii at iteration ( $k$ ). If this difference exceeds a specified tolerance value ( $\epsilon$ ), the iterative process continues.

In summary, the IIFDPF algorithm follows a specific updating strategy for voltage angle and voltage magnitude at each bus, and the convergence is evaluated based on the difference between radii in consecutive iterations. By controlling the tolerance value, the algorithm determines when to terminate the iterative process and achieve convergence.

### III. RESULT AND DISCUSSION

In this section, we apply the proposed methodology to perform power flow analysis considering uncertainties in the IEEE 30, 57, and 118 bus test systems. The objective is to determine power flow solution bounds using an Interval Analysis (IA)-based technique and compare them with the bounds obtained through Monte Carlo simulation using a uniform distribution. To incorporate uncertainties in the input data, we assume a 10% uncertainty on the load and generator power bounds, as illustrated in Figure 1(a) and Figure 1(b). We then employ the proposed IA-based methodology to calculate the power flow solution bounds considering these uncertainties. The IA-based technique allows us to obtain robust bounds for the power flow solutions. By comparing the bounds obtained through IA-based calculations with those obtained using Monte Carlo simulation, we can evaluate the effectiveness of the proposed methodology in capturing the uncertainties and providing reliable power flow solution bounds.

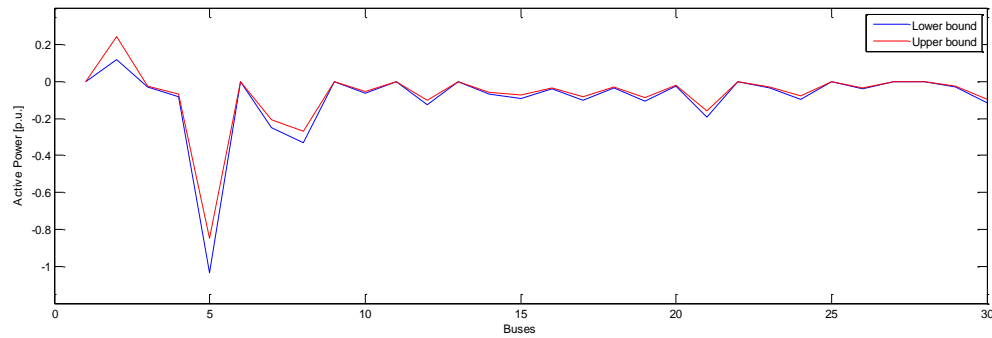


Fig.1 (a). Assumed Uncertain Active power with 10% uncertain error in input data

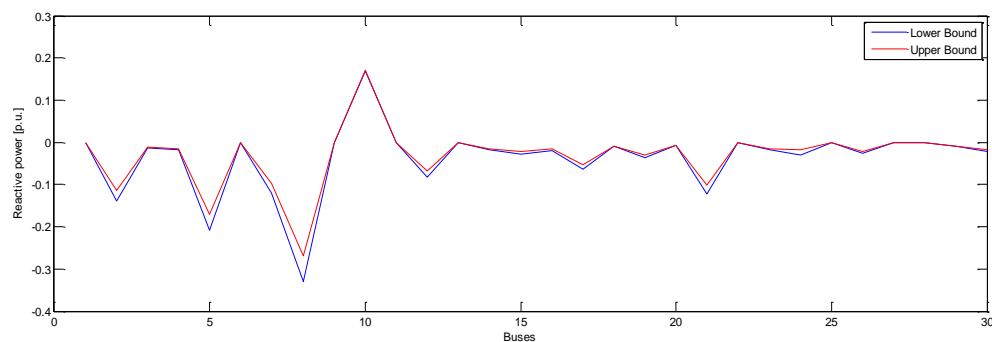


Fig.1(b). Assumed Uncertain Reactive power with 10% uncertain error in input data

The performance of the IIFDPFL (Interval Interval Fast Decoupled Power Flow) algorithm was evaluated on IEEE 30, 57, and 118 bus systems, considering a convergence accuracy of  $10^{-3}$  times on a MVA (Mega Volt-Ampere) base of 100 for both power residuals, delta P and delta Q.

In this paper, the proposed algorithm introduced a strategy where the voltage angle is updated multiple times before updating the voltage magnitude, or vice versa. This approach resulted in different convergence speeds depending on the combination of variable updates employed for each IEEE bus system. These combinations are denoted based on the number of loops for updating each variable. For example, updating the voltage magnitude (V) twice and then updating the voltage angle ( $\delta$ ) once within the same iteration is denoted as (2:1).

Additionally, the study investigated various combinations of updating the voltages and phase angles to analyze their effects on computational efficiency and convergence speed under uncertainty. By exploring different update combinations, the algorithm aimed to achieve computational savings and faster convergence, particularly in the presence of uncertainty. By examining the impact of different variable update strategies, the research aimed to optimize the performance of the IIFDPFL algorithm, enabling faster convergence and improved computational efficiency when dealing with uncertain power systems.

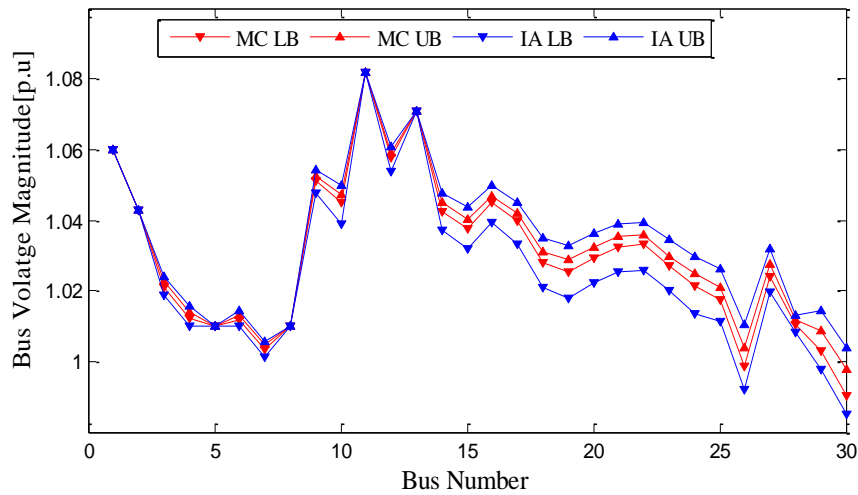


Fig.2. Interval Voltage magnitude of IEEE 30bus system

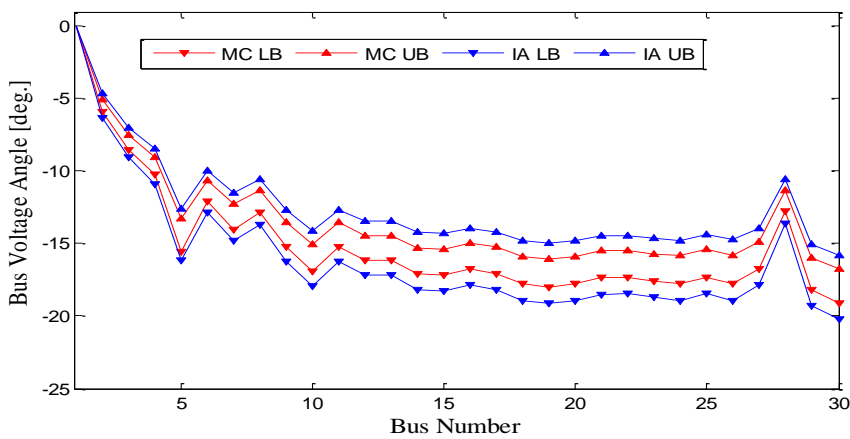


Fig.3. Interval Voltage Angle of IEEE 30bus system

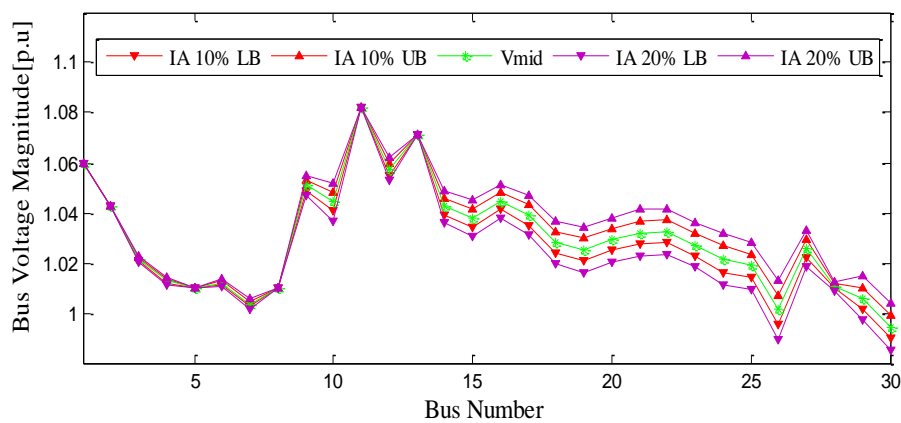


Fig.4. Interval Voltage magnitude with conventional voltage mid with 10% and 20% uncertain error

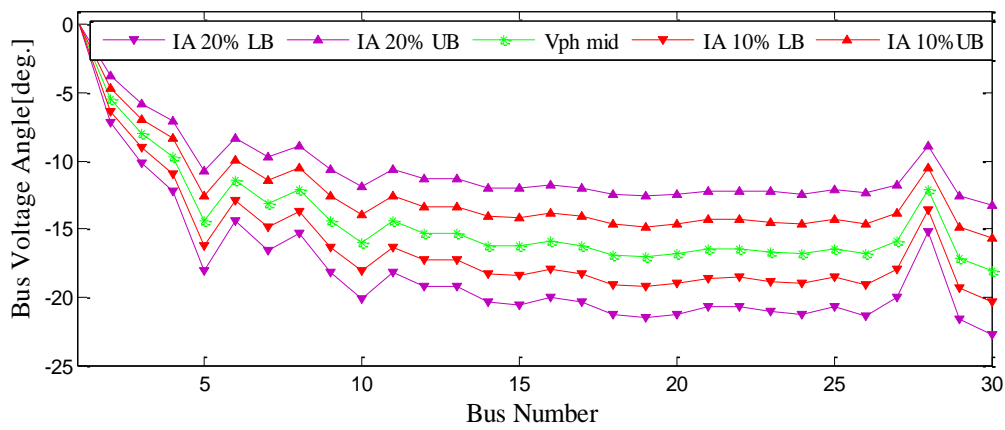


Fig.5.Interval Voltage angle with conventional voltage angle mid with 10% and 20% uncertain error

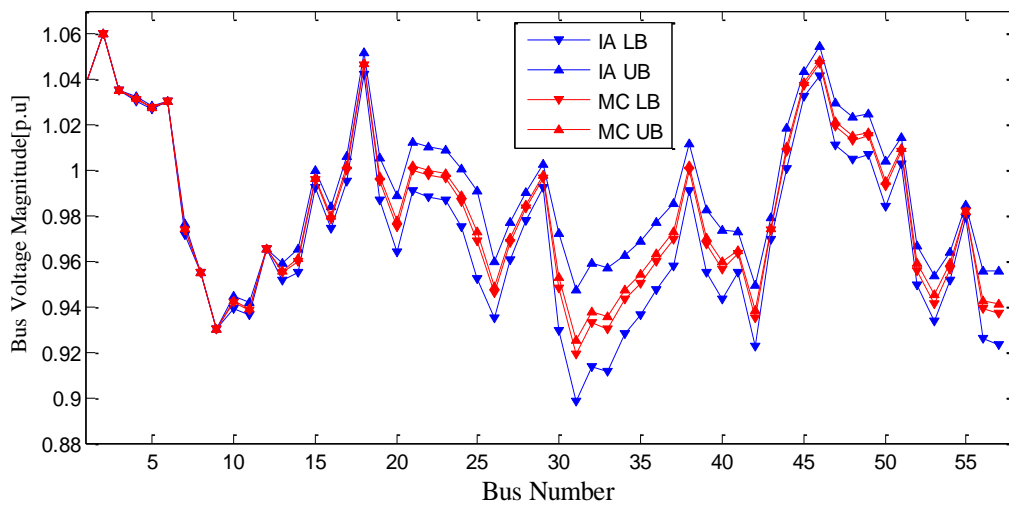


Fig.6.Interval Voltage magnitude with 10% uncertain error of IEEE 57 bus system

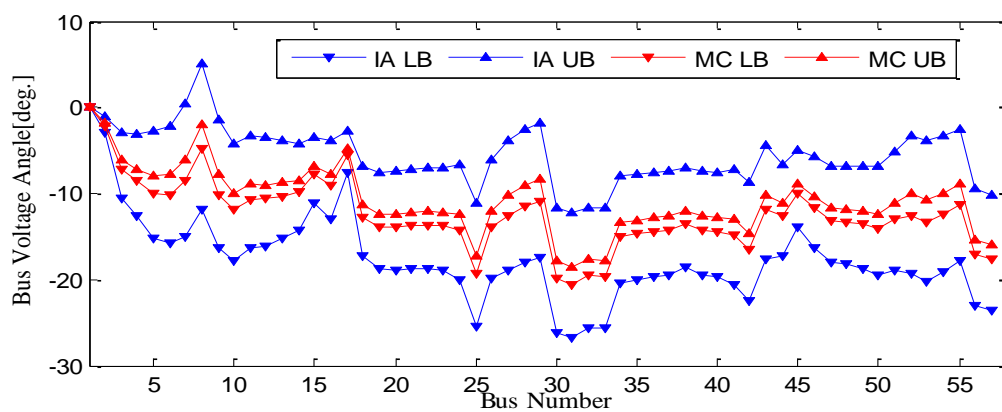


Fig.7.Interval Voltage angle with 10% uncertain error of IEEE 57 bus system

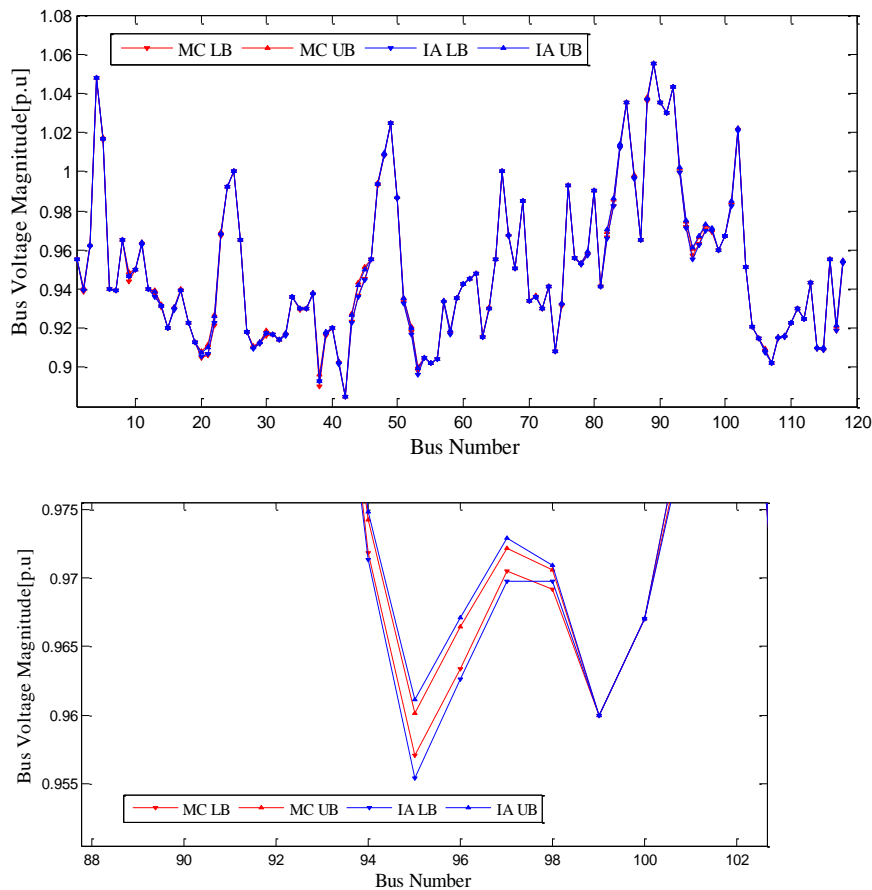


Fig.8.Interval Voltage Magnitude with 10% uncertain error of IEEE 118 bus system

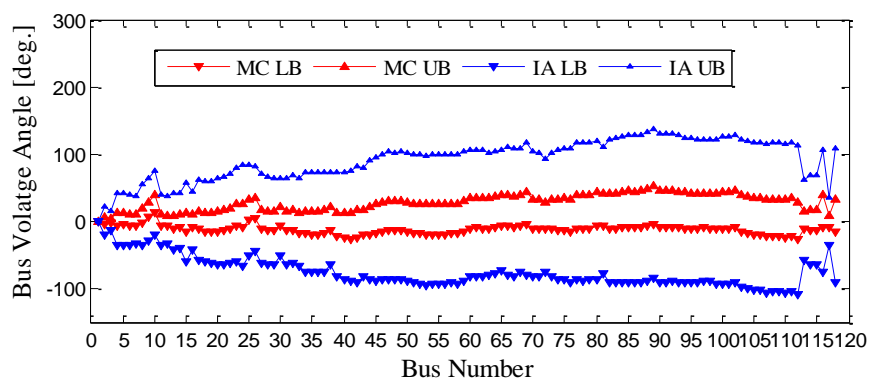


Fig.9.Interval Voltage angle with 10% uncertain error of IEEE 118 bus system

In order to validate proposed model with used a known probabilistic method based on Monte Carlo simulation. As it show from the Fig.2 to Fig.9 shows the results at 10% and 20% of uncertainty interval results enclosed punctual value and 10% of uncertainty, the result from interval arithmetic is more conservative than that of Monte Carlo approach and the system converges in three iterations for both cases.

MC method taken 1000 iterations to converges, whereas proposed IIFDPF method converged in three iterations for voltage and voltage angle updated in 2:1(i.e. two time voltage and one time phase angle).

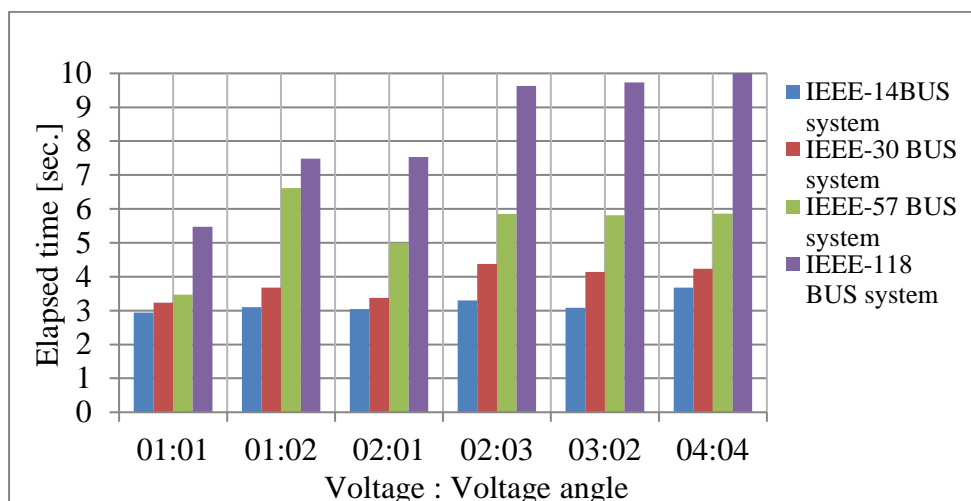


Fig.10 : Time (sec) taken for different strategic of updating voltage and voltage angle (Voltages (V): Phase angle ( $\delta$ ))

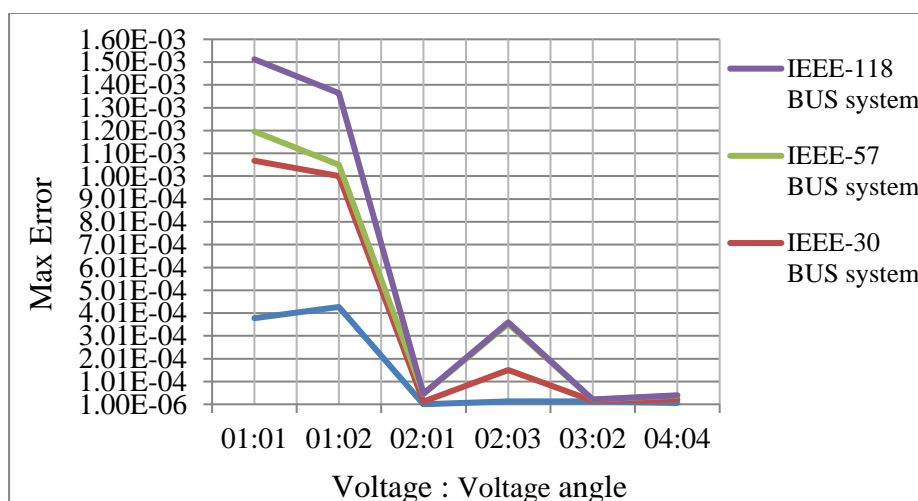


Fig.11 : Maximum error taken for different strategic of updating voltage and voltage angle

Table 1: Number of iteration, Power mismatch and Elapsed time

Voltage Phase angle		01:01	01:02	02:01	02:03	03:02	04:04
IEEE-30 BUS system	Elapsed time	3.23	3.68	3.37	4.37	4.14	4.23
	Max. error	6.91E-04	5.74E-04	1.16E-05	1.37E-04	2.63E-06	1.57E-05

<b>IEEE-57 BUS system</b>	<b>Elapsed time</b>	3.46	6.61	5.00	5.85	5.81	5.86
	<b>Max. error</b>	1.28E-04	4.89E-05	3.28E-05	2.03E-04	4.76E-06	1.70E-05
<b>IEEE-118 BUS system</b>	<b>Elapsed time</b>	6.04	7.48	7.53	9.63	9.73	10.25
	<b>Max. error</b>	3.16E-04	3.14E-04	1.08E-06	6.56E-06	6.37E-07	6.27E-07

From the table 1 and the figure 10 and 11, different values of maximum error and elapsed time of simulation for different voltage and voltage angle update. In the first case for IEEE-14 bus system using ten percent of error i.e. uncertainty iteration the case of 01:01 elapsed time is 2.93sec. and max error is 1.20E-06 which is the smallest of all. But for IEEE-30, IEEE-57 and IEEE-118 bus systems the case of 2:1 has an error of 1.16E-05, 3.28E-05 and 1.08E-06 respectively. When the number of buses increases the case of 2:1 has less error and fast convergence in comparison with the other.

## CONCLUSIONS

The analysis presented highlights various methods employed to address the challenges posed by nonlinear load flow problems with uncertain solutions. However, there is still scope for further research to explore and develop more reliable techniques for solving power flow problems while considering uncertainties in load and generator conditions. One promising avenue for investigation is the utilization of interval mathematics. In the proposed algorithm, the voltage angle is updated multiple times before updating the voltage magnitude, or vice versa, resulting in different convergence speeds for different combinations within the same IEEE bus system. This approach has been validated against Monte Carlo simulation, and the results demonstrate that interval methods exhibit computational superiority over traditional Monte Carlo simulations. To enhance the reliability of power flow solutions in the presence of uncertainties, future research efforts can focus on refining the proposed methodology and exploring alternative approaches based on interval mathematics. These methods have the potential to provide more accurate and robust solutions, leading to improved analysis and decision-making in power system operations.

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